

# Open problems for FPT School 2014

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This list contains a compilation of open problems from recent Dagstuhl Seminars or Workshop on Kernels, as well as some problems mentioned in some recent papers. We tried to rank them (with stars) depending on their importance and possible hardness, but please do not take the ratings too seriously.

**Change log:**

21 Aug 2014    open problem list made public  
1 Sep 2014    Knapsack problem reported to be solved

## Even Set aka Minimum Codeword (★ ★ ★)

*Long standing; appeared, e.g., in [37].*

In the EVEN SET problem the input consists of a family  $\mathcal{F}$  of subset of a universe  $U$  and an integer  $k$ ; the question is to find a nonempty set  $A \subseteq U$  of size at most  $k$  such that  $|A \cap F|$  is even for every  $F \in \mathcal{F}$ . Alternatively, the question can be stated as finding a non-zero codeword of Hamming weight at most  $k$  in a linear code over  $\mathbb{F}_2$ . The question of parameterized complexity of this problem, parameterized by  $k$ , remains open.

Note that if we require the set  $A$  to be of size exactly  $k$ , or we require the intersections to be odd, the problem becomes  $W[1]$ -hard.

## Framework for refuting Turing kernels (★ ★ ★)

*Long-standing, appeared, e.g., in [37, 27].*

One of the most important open problems in kernelization is to provide a framework for refuting Turing kernels. Currently, we know that there is a large group of problems equivalently (un)likely to have Turing kernels [48]. An interesting example of a Turing kernel appears in [52].

## Tight bounds for kernels for Vertex Cover (★ ★ ★)

*Long-standing; appeared, e.g., in [27].*

It seems reasonable to believe that the  $2k$ -vertex kernel for VERTEX COVER [69] is optimal, as a  $(2 - \varepsilon)$ -approximation algorithm for VERTEX COVER would violate the Unique Games Conjecture [53], and it is hard to imagine a  $(2 - \varepsilon)k$ -vertex kernel that would not yield a  $(2 - \varepsilon')$ -approximation algorithm for VERTEX COVER. However, the aforementioned argumentation is informal, and there exists an example of a problem with a polynomial kernel, but without matching approximation algorithm [44]. Can we prove a matching lower bound for the  $2k$ -vertex kernel, assuming some widely-believed complexity assumption?

A similar question can be considered in the case of planar graphs. Here, no approximation arguments restrict us, as VERTEX COVER admits a PTAS in planar graphs (via the classical Baker’s approach [5]). Note that the  $4k$ -vertex kernel for INDEPENDENT SET in planar graphs yields an  $(\frac{4}{3} - \varepsilon)k$ -vertex lower bound for a VERTEX COVER kernel in planar graphs [19]. However, still the best known upper bound is the  $2k$ -vertex kernel inherited from general graphs.

## A linear element-kernel for $d$ -Hitting Set (★★)

*Appeared in [27].*

The VERTEX COVER problem admits a  $2k$ -vertex kernel [69], but is unlikely to admit a  $\mathcal{O}(k^{2-\varepsilon})$ -edge kernel [35]. More generally, we know that the  $d$ -HITTING SET admits a kernel with  $\mathcal{O}(k^d)$  sets and  $\mathcal{O}(k^{d-1})$  elements [1], and a matching lower bound for the number of sets is known [35]. However, it remains open whether we can further reduce the number of elements in the kernel. In particular, does  $d$ -HITTING SET admit a kernel with  $f(d)k$  vertices?

## Polynomial kernel for Imbalance (★)

*Appeared in [37].*

Let  $G$  be an  $n$ -vertex graph. For an ordering  $\sigma : V(G) \rightarrow \{1, 2, \dots, n\}$ , the imbalance of a vertex  $v$  equals

$$I(\sigma, v) = ||\{u \in N_G(v) : \sigma(u) < \sigma(v)\}| - |\{u \in N_G(v) : \sigma(u) > \sigma(v)\}||.$$

The imbalance of  $\sigma$  is defined as  $I(\sigma) = \sum_{v \in V(G)} I(\sigma, v)$ . Although it is relatively easy to obtain an FPT algorithm for finding an ordering of imbalance at most  $k$  (parameterized by  $k$ ) [58], the question of polynomial kernel remains open.

## Cutting short paths (★)

*From [45].*

In [45] the authors study (among others) the following problem: given a (directed or undirected) graph  $G$  with source  $s$  and sink  $t$ , and integers  $k$  and  $l$ , cut at most  $k$  edges of  $G$  so that a shortest path from  $s$  to  $t$  is of length larger than  $l$ . They show an FPT algorithm, parameterized by both  $k$  and  $l$ . The question is: does this problem admit a polynomial kernel with respect to this parameterization?

## Improving branching algorithms for some classic problem (★★)

*Based on [20, 40, 39, 51].*

The following algorithms are current champions for classic problems:

- VERTEX COVER: fastest FPT is  $\mathcal{O}(1.2738^k + kn)$  [20], whereas if we parameterize by the number of vertices the champion is  $\mathcal{O}^*(2^{0.288n}) \leq \mathcal{O}^*(1.23^n)$  [40].
- FEEDBACK VERTEX SET, parameterized by the number of vertices:  $\mathcal{O}^*(1.7548^n)$  [39].
- DOMINATING SET, parameterized by the number of vertices:  $\mathcal{O}^*(1.4689^n)$  in exponential space and  $\mathcal{O}^*(1.4864^n)$  in polynomial space [51].

## Faster Subset Sum (★ ★ ★)

*Long-standing, see [4].*

In the SUBSET SUM problem we are given  $n$  integers  $x_1, x_2, \dots, x_n$  and an integer  $M$  and we ask if there exists a set  $S \subseteq \{1, 2, \dots, n\}$  such that  $\sum_{i \in S} x_i = M$ . A brute-force algorithm solves this problem in  $\mathcal{O}^*(2^n)$  time and polynomial space, whereas a simple meet-in-the-middle approach gives  $\mathcal{O}^*(2^{n/2})$  time and space. Can any of these bounds be exponentially improved? That is, we ask for an  $\mathcal{O}(c^n)$ -time algorithm with polynomial space for some  $c < 2$  or an  $\mathcal{O}(c^{n/2})$ -time algorithm for some  $c < 2$  that may use exponential space.

## Faster exponential 3-coloring (★)

*Based on [6].*

The fastest known algorithm to check if the input graph is 3-colorable runs in  $\mathcal{O}(1.3289^n)$  time [6]. Can it be substantially improved?

## Game of Kayles (★)

*Based on [12].*

In the Kayles game the board is an input graph, and two players build an independent set in the graph; in a single move, a player adds a new vertex to the independent set. The player that cannot insert any new vertex (i.e., the constructed independent set is an inclusion-wise maximal) loses. In the Kayles game problem, given a graph  $G$ , we ask which player has a winning strategy in this game. This is one of the classic PSPACE-complete problems [75], and we can easily solve it in time and space  $\mathcal{O}^*(2^n)$  (because this is the number of states in the game). Keeping the use of exponential space, it can be improved to  $\mathcal{O}(1.6052^n)$  [12].

- Can the Kayles game be solved in time  $\mathcal{O}(c^n)$  and polynomial space for some  $c < 2$ ?
- What is the complexity of this problem if  $G$  is a tree? No hardness result is known, and the fastest known algorithm is exponential [12].

## Number of minimal dominating sets in a graph (★)

*Based on [41].*

How many inclusion-wise minimal dominating sets can be contained in an  $n$ -vertex graph? The best known lower bound is  $15^{n/6} < 1.5705^n$  (a disjoint union of graphs isomorphic to  $K_6$  with a perfect matching removed), while the best known upper bound is  $\mathcal{O}(1.7159^n)$  [41].

## Faster FPT algorithm for Feedback Vertex Set (★★)

*Based on [33] and [55].*

The fastest known FPT algorithms for FEEDBACK VERTEX SET run in  $\mathcal{O}^*(3^k)$  randomized time [33] and  $\mathcal{O}^*(3.62^k)$  deterministic time [55]. Can they be improved? In particular, we expect that it should be possible to obtain an  $\mathcal{O}^*(3^k)$ -time deterministic algorithm for the problem. Note that a similar result has been obtained for CONNECTED VERTEX COVER [25].

We remark here that the authors of [55] in their technical report [54] observed that it is relatively easy (but very tedious) to improve slightly the base of the exponent, at the cost of very extensive case analysis. This is not an improvement we are looking for.

## Faster FPT algorithm for Eulerian Edge Deletion (★)

*Mentioned in [32].*

In the EULERIAN EDGE DELETION problem, given a graph  $G$ , we ask to remove at most  $k$  edges to obtain a graph with an Eulerian tour. In [32] a  $2^{\mathcal{O}(k \log k)} n^{\mathcal{O}(1)}$ -time algorithm is presented. Can the dependency on  $k$  be improved to  $2^{\mathcal{O}(k)}$ ?

## Longest common subsequence for strings with arcs (★)

*From [64].*

A string with arcs is a string over some finite (fixed, constant-size) alphabet  $\Sigma$ , where additionally some pairs of letters are connected by edges, and every letter is incident to at most one edge. Given two strings with arcs  $A$  and  $B$ , and integers  $k_A$  and  $k_B$ , we would like to delete  $k_A$  letters from  $A$  and  $k_B$  letters from  $B$  to obtain equal words (including the edges). Is this problem FPT, parameterized by  $k_A + k_B$ ? In [64] it is shown that it is FPT for the restricted case  $k_B = 0$ .

## Faster algorithms for TSP and related problems (★★★)

*Appeared in [49].*

The problem of finding minimum/maximum cost Hamiltonian cycle can be solved in  $\mathcal{O}^*(2^n)$  time by a standard dynamic programming algorithm. Can it be solved in  $\mathcal{O}(c^n)$  time for some  $c < 2$ ? The existence of a Hamiltonian cycle in undirected graphs can be detected in  $\mathcal{O}(1.66^n)$  time [8], but the directed case is open (but the parity of the number of such cycles can be found quicker [9]). Also, even for a possibly simpler case of SHORTEST SUPERSTRING (given  $n$  strings, what is the shortest string that contains every input string as a subword) we do not know a faster algorithm than  $\mathcal{O}^*(2^n)$ .

## Faster algorithm for Maximum Acyclic Subgraph (★)

*Appeared in [49].*

Given a directed graph  $G$ , the MAXIMUM ACYCLIC SUBGRAPH problem asks for an acyclic subgraph with maximum possible number of edges. A simple dynamic programming algorithm solves this problem in  $\mathcal{O}^*(2^n)$  time for  $n$ -vertex graphs. Is it possible to obtain a  $\mathcal{O}(c^n)$ -time algorithm for some  $c < 2$ ?

Note that for the “induced subgraph” problem, where we maximize the number of vertices in the subgraph, the problem is equivalent to DIRECTED FEEDBACK VERTEX SET, and the answer is positive [73].

## Faster algorithm for Cutwidth (★)

*Appeared in [49] and in [31].*

The cutwidth of a graph  $G$  is the smallest integer  $k$  such that the vertices of  $G$  can be arranged in a linear layout  $v_1, v_2, \dots, v_n$  such that for every  $1 \leq i < n$  there are at most  $k$  edges between  $\{v_1, v_2, \dots, v_i\}$  and  $\{v_{i+1}, v_{i+2}, \dots, v_n\}$ . A simple dynamic programming algorithm finds cutwidth of an  $n$ -vertex graph in time  $\mathcal{O}^*(2^n)$ . Does there exist an algorithm running in time  $\mathcal{O}(c^n)$  for some  $c < 2$ ?

Note that the problem can be solved in time  $2^k n^{\mathcal{O}(1)}$  for graphs with vertex cover of size at most  $k$  [31].

## Planar Independent Set above guarantee (★★)

*Appeared in [37, 13].*

A direct consequence of the Four Colour Theorem is that every  $n$ -vertex planar graph admits an independent set of size at least  $n/4$ . Consider the following question: given an  $n$ -vertex planar graph  $G$  and an integer  $k$ , we ask if  $G$  admits an independent set of size at least  $(n+k)/4$ . Is this problem FPT when parameterized by  $k$ ?

A related question of being “above guarantee” for the triangle-free planar graphs has been resolved positively in [36].

## Subexponential algorithms for planar problems (★★)

*Appeared in [13].*

Despite the robustness of the bidimensionality framework, for a few planar problems we still do not know whether they admit a subexponential algorithm. This includes:

- LONGEST PATH in directed planar graphs;
- WEIGHTED  $k$ -PATH in undirected graphs (maximum weight path on  $k$  vertices);
- EXACT  $k$ -CYCLE (does there exist a cycle on exactly  $k$  vertices);
- STEINER TREE, parameterized by the number of terminals;
- SUBGRAPH ISOMORPHISM, parameterized by the size of the pattern graph.

## Subgraph isomorphism in planar graphs parameterized by the difference (★)

*Appeared in [13].*

Consider a SUBGRAPH ISOMORPHISM problem, parameterized by the difference  $|E(G)| - |E(H)|$ . Is it fixed-parameter tractable on planar graphs? Recall that the GRAPH ISOMORPHISM problem on planar graphs is polynomial, due to (a) uniqueness of the embedding of 3-connected planar graphs, and (b) uniqueness of the Tutte’s decomposition into 3-connected components.

## Parameterized Complexity of Directed Multicut (★★)

*Appeared in [26].*

Is DIRECTED MULTICUT fixed-parameter tractable when parameterized by the number of terminals and the size of the cutset? We know that:

1. DIRECTED MULTICUT is FPT when parameterized by the size of the cutset only [24].
2. DIRECTED MULTICUT is W[1]-hard when parameterized by the size of the cutset only [63], even in DAGs [56].
3. DIRECTED MULTICUT is FPT when parameterized by the size of the cutset and the number of terminals, when the input graph is a DAG [56].
4. DIRECTED MULTICUT is NP-hard and APX-hard for two terminal pairs [7], but the two-terminal case can be reduced to DIRECTED MULTICUT. It is open whether it is FPT for 3 terminal pairs, parameterized by the size of the cutset.

## Faster algorithms for finding good cuts (★)

Appeared in [26]. Further discussion in [30].

A  $(q, k)$ -good cut in an undirected graph  $G$  is a set  $X$  of at most  $k$  edges, such that  $G \setminus X$  has exactly two connected components, each containing more than  $q$  vertices. The notion of good cuts is essential for recursive calls in the  $k$ -WAY CUT algorithm by Kawarabayashi and Thorup [50] and in the follow-up technique of randomized contractions [22]. Via randomized contractions, a  $(q, k)$ -good cut can be found in roughly  $\mathcal{O}^*(q^k)$  time. Can this running time be significantly improved? A positive answer would be a first step to speed up the algorithms based on the randomized contractions technique, which currently seem to be stuck at running time  $\mathcal{O}^*(2^{\mathcal{O}(k^2 \text{polylog}(k))})$ . Also, it has a potential to improve the running time of the recent decomposition used for MINIMUM BISECTION [30].

## Faster algorithms for Odd Cycle Transversal and related problems (★)

Appeared in [26].

The currently fastest FPT algorithm for ODD CYCLE TRANSVERSAL and VERTEX COVER ABOVE LP runs in  $\mathcal{O}^*(2.318^k)$  time [67, 59]. The base of the exponent comes from branching vectors with complicated case analysis, so we do not expect it to be optimal. Can it be significantly improved? For example, to  $\mathcal{O}^*(2^k)$ ?

A related question is to improve the  $\mathcal{O}^*(2^k)$  algorithm for EDGE BIPARTIZATION [46].

## Parameterized complexity of König Edge Deletion (★)

Appeared in [26].

An undirected graph is a König graph if it admits a vertex cover of size equal to the size of its maximum matching. This class contains all bipartite graphs, but not every König graph is bipartite; for example, a triangle with a pendant vertex attached to one vertex is a König graph (see e.g. [66]).

Consider the KÖNIG EDGE DELETION problem where we are to delete as few edges as possible from the given graph to obtain a König graph. Is this problem FPT, parameterized by the number of edge deletions? Note that this problem is at least as hard as ALMOST 2-SAT, and the vertex deletion variant is shown to be FPT in [59].

## A single-exponential algorithm for Directed FVS (★★)

Appeared in [26].

Since 2008 we know that DIRECTED FEEDBACK VERTEX SET is fixed-parameter tractable, but the only known algorithm runs in  $\mathcal{O}^*(k!4^k)$  time [21]. The  $k!$  factor comes out from considering all orderings of the modulator set in the iterative compression step; the rest of the algorithm runs in  $\mathcal{O}^*(2^{\mathcal{O}(k)})$  time. Can this step be avoided, so that DFVS would be solved in  $\mathcal{O}^*(2^{\mathcal{O}(k)})$  time? Or maybe it is impossible, assuming ETH?

## Parameterized complexity of Stable Multicut (★)

Appeared in [26].

Since 2011 we know that MULTICUT, parameterized by the size of the cutset, is fixed-parameter tractable [14, 63]. What is the parameterized complexity of the variant of this problem, when we require the cutset to

be independent? Note that, when parameterized by the size of the cutset and the number of terminal pairs, the problem is FPT due to the treewidth reduction technique [61, 62].

## Polynomial kernel for Directed FVS (★ ★ ★)

*Long-standing; appeared, e.g., in [37, 27].*

One of the long-standing open problems is the question of an existence of a polynomial kernel for DIRECTED FEEDBACK VERTEX SET, parameterized by the size of the deletion set. The FPT algorithm is known since 2008 [21].

## Polynomial kernel for Multiway Cut (★★)

*Long-standing; appeared, e.g., in [27].*

The recent applications of matroid techniques to kernelization resulted in a  $\mathcal{O}(k^{t+1})$ -vertex kernel for MULTIWAY CUT with  $t$  terminals and  $k$  being the bound on the size of the cutset [57]. Can the dependency on  $t$  be removed from the exponent? The problem remains open even in the (easier) edge-deletion variant of MULTIWAY CUT.

The question has been resolved positively in the planar case [71], but with extremely big exponent.

## Polynomial kernel for Multicut in DAGs (★)

*From [56, 29], appeared also in [27].*

In [29] the authors refute the existence of polynomial kernels for most graph separation problems in directed graphs, as DIRECTED MULTIWAY CUT with 2 terminals is OR-compositional. The remaining case is the MULTICUT problem in directed acyclic graphs (shown to be FPT in [56]). Does it admit a polynomial kernel, when parameterized by the size of the cutset and the number of terminal pairs? Or when parameterized by the size of the cutset, with constant number of terminal pairs?

## Eulerian SCC Deletion (★)

*Appeared originally in [18], asked in [32, 27].*

In the EULERIAN SCC DELETION problem, given a directed graph  $G$  and an integer  $k$ , we ask whether it is possible to delete at most  $k$  arcs from  $G$  to obtain a graph where each strongly connected component contains an Euler tour. Is EULERIAN SCC DELETION fixed-parameter tractable, when parameterized by  $k$ ?

A few remarks are in place. The question of fixed-parameter tractability of EULERIAN SCC DELETION was originally posted by Cechlárová and Schlotter in [18], where it appeared naturally in modelling of housing markets. A somehow related deletion problems were studied in [32]. However, it is not hard to reduce DIRECTED FEEDBACK VERTEX SET to EULERIAN SCC DELETION, and, hence, we expect that a hypothetical fixed-parameter algorithm for EULERIAN SCC DELETION would need to use substantially different techniques than the ones developed in [32].

## Chain SAT (★)

*Asked in [23].*

In the  $\ell$ -CHAIN SAT problem we are given a set of  $n$  Boolean variables, a set of constraints of the form  $x_1 \Rightarrow x_2 \Rightarrow \dots \Rightarrow x_r$  where  $r \leq \ell$ , and an integer  $k$ . The question is to delete at most  $k$  constraints to obtain

a satisfiable instance. For a fixed integer  $\ell$ , does this problem admit an FPT algorithm, parameterized by  $k$ ?

## **[SOLVED] ~~A polynomial kernel for Knapsack (\*\*)~~**

*Appeared in [27].*

**Reported to be solvable using a result of [43], similarly to how it is used in [65].**

In the KNAPSACK problem, we are given  $n$  items with sizes  $(s_i)_{i=1}^n$  and values  $(v_i)_{i=1}^n$  and capacity of the knapsack  $B$ ; we are to choose a set of items  $A \subseteq \{1, 2, \dots, n\}$  that fit into the knapsack ( $\sum_{i \in A} s_i \leq B$ ) and have maximum possible total value (maximize  $\sum_{i \in A} v_i$ ). Does this problem admit a polynomial kernel with respect to parameter  $n$ ? That is, can we reduce the sizes and the values, so that their bit-length is bounded polynomially in  $n$ ?

The answer is affirmative (using randomization) for a related problem of SUBSET SUM [47]. Moreover, there exists a randomized Turing kernel for KNAPSACK parameterized by  $n$  [68]. More formally, the algorithm of [68] outputs  $\ell$  KNAPSACK instances such that

1. the answer to the original instance is an OR of the output instances;
2. the algorithm is randomized with one-sided error (it may produce false positives);
3.  $\ell$  is bounded polynomially in  $n$  and the bit-length of the input sizes and values; and
4. each output size and value have bit-length bounded polynomially in  $n$ .

## **Max-leaf outbranching, parameterized by treewidth (\*)**

*Based on [33].*

In a directed graph, an outbranching is a subgraph that is a rooted tree, where each arc is directed downwards. In the MAX-LEAF OUTBRANCHING problem we seek for an outbranching in the given graph with maximum number of leaves. We are interested in solving this problem, when we are given a tree decomposition of  $G$  of width  $t$ , that is, we study treewidth DPs. In [33] it is shown how to make a Cut&Count-based algorithm running in time  $\mathcal{O}^*(6^t)$ , but no matching lower bound is shown (contrary to most problems there). Is 6 the optimal base of the exponent? (Of course, assuming Strong ETH). Or maybe you can do better?

## **Faster algorithm for computing chromatic number (\*\*\*)**

*Based on [10].*

With the use of Fast Subset Convolution or the inclusion-exclusion principle we can compute the chromatic number of an  $n$ -vertex graph in time and space  $\mathcal{O}^*(2^n)$  or in time  $\mathcal{O}^*(2.246^n)$  and polynomial space [10]. Can any of the running time bounds be improved? In particular, can we compute chromatic number in  $\mathcal{O}^*(2^n)$  time and polynomial space?

## **Subgraph isomorphism in $c^n$ time (\*\*\*)**

*Appeared in [49] and discussed in [2].*

Can the subgraph isomorphism of two  $n$ -vertex graphs be solved in  $\mathcal{O}(c^n)$  time for some constant  $c$ ?

## **Chromatic index in $c^n$ time (\*\*)**

*Folklore.*



The chromatic index of a graph is the minimum number of colors in which we can color the edges of the graph such that no two edges with a common endpoint have the same color. The classic theorem of Vizing asserts that the chromatic index of a graph is either  $\Delta$  or  $\Delta + 1$ , where  $\Delta$  is the maximum degree. However, detecting which is the case is NP-hard. Moreover, it is open whether it can be done in  $\mathcal{O}(c^n)$  time on  $n$ -vertex graphs, for some constant  $c$ .

## A different convolution (★)

Based on [28].

Given two functions  $f, g : 2^U \rightarrow \mathbb{Z}$ , we would like to compute the following convolution in time  $\mathcal{O}^*(2^{|U|})$ :

$$(f \diamond g)(X) = \sum_{\substack{A \cup B = X \\ A \cap B = \emptyset}} f(A)g(B)(-1)^{\text{inv}(A,B)},$$

where  $\text{inv}(A, B) = |\{(a, b) : a \in A, b \in B, a < b\}|$ , assuming some fixed total order on  $U$ . An  $\mathcal{O}^*(2^{|U|})$ -time algorithm for the above convolution may lead to faster algorithm for counting Hamilton cycles, parameterized by treewidth [28].

## Line Graph Edge Deletion (★)

Appeared in [27].

The LINE GRAPH EDGE DELETION problem asks to delete at most  $k$  edges from the input graph to obtain a line graph. The characterization by forbidden induced subgraphs yields a  $\mathcal{O}^*(11^k)$ -time algorithm. Can this algorithm be significantly improved? Does this problem admit a polynomial kernel?

## Claw-free Edge Deletion (★)

Discussed in [15, 16]. Appeared in [27].

A similar question as before can be asked for the CLAW-FREE EDGE DELETION problem. A graph is claw-free if it does not contain a  $K_{1,3}$  as an induced subgraph, and every line graph is a claw-free graph. The forbidden induced subgraphs characterization immediately yields a  $\mathcal{O}^*(3^k)$  FPT algorithm. What about a polynomial kernel?

Recent progress in similar problems include [15] (also in thesis form [16]) and [3]. The thesis [16] contains a detailed discussion of remaining open cases of kernelization of  $H$ -FREE EDGE DELETION problems.

## Polynomial kernels for interval/chordal modification problems (★)

Appeared in [27].

There are more graph edition problems where a question of a polynomial kernel is open.

1. INTERVAL VERTEX DELETION, shown recently to be FPT [17, 72].
2. CHORDAL VERTEX DELETION [60].
3. INTERVAL COMPLETION [77, 11].
4. PROPER INTERVAL VERTEX DELETION: there is a  $\mathcal{O}(k^{53})$  kernel [42] and a  $\mathcal{O}^*(6^k)$  FPT algorithm [76]. Can we obtain a significantly smaller kernel with, say, at most  $\mathcal{O}(k^{10})$  vertices?

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